Journal of Engineering Physics and Thermophysics, Vol. 82, No. 4, 2009

ELECTRODYNAMICS OF LAYERED MEDIA WITH BOUNDARY CONDITIONS CORRESPONDING TO THE TOTAL-CURRENT CONTINUUM

N. N. Grinchik,[a] **A. P. Dostanko,**[b]
I. A. Gishkelyuk,[c] **and Yu. N. Grinchik**[d]

UDC 541.131+519.6+539.217.1

A consistent physical and mathematical model of propagation of electromagnetic waves in layered media, in which the induced surface charge is not explicitly taken into account, has been constructed on the basis of Maxwell equations, the total-current continuum equation, and the Dirichlet theorem. A numerical method of through counting for solving the system of equations defining layered media has been developed and substantiated. Examples of numerical simulation of the propagation of an electromagnetic wave in a two-dimensional cellular structure and the propagation of a modulated signal in a one-dimensional layered medium on condition that the current flows at the interfaces between the adjacent media are dependent are presented.

Keywords: *electromagnetic waves, layered media, Maxwell equations, physical and mathematical model.*

Introduction. Let us consider the interface S between two media having different electrophysical properties. On each of its side the magnetic-field and magnetic-inductance vectors as well as the electric-field and electric-displacement vectors are finite and continuous; however, at the surface S they can experience a discontinuity of the first kind. Moreover, at the interface there arise induced surface charges σ and surface currents i (whose vectors lie in the plane tangential to the surface S) under the action of an external electric field.

The existence of a surface charge at the interface S between the two media having different electrophysical properties is dramatically illustrated by the following example. We will consider the traverse of a direct current through a flat capacitor filled with two dielectric materials having relative permittivities ε_1 and ε_2 and electric conductivities λ_1 and λ_2. A direct-current voltage U is applied to the capacitor plates; the total resistance of the capacitor is R (Fig. 1). It is proposed to calculate the surface electric charge induced by the electric current.

From the electric-charge conservation law follows the constancy of flow in a circuit; therefore, the following equation is fulfilled:

$$\lambda_1 E_{n_1} = \lambda_2 E_{n_2} = U/(RS)\,, \tag{1}$$

where E_{n_1} and E_{n_2} are the normal components of the electric-field vector.

At the interface between the dielectrics the normal components of the electric-inductance vector change spasmodically under the action of the electric field by a value equal to the value of the induced surface charge σ:

$$\varepsilon_0\varepsilon_1 E_{n_1} - \varepsilon_0\varepsilon_2 E_{n_2} = \sigma\,. \tag{2}$$

Solving the system of Eqs. (1), (2), we obtain the expression for σ

$$\sigma = (U/RS)\,\varepsilon_0\,[(\varepsilon_1/\lambda_1) - (\varepsilon_2/\lambda_2)]\,. \tag{3}$$

[a]A. V. Luikov Heat and Mass Transfer Institute, National Academy of Sciences of Belarus, 15 P. Brovka Str., Minsk, 220072, Belarus; email: nngrin@yandex.ru; [b]Belarusian State Institute of Information Science and Radio Electronics, 6 P. Brovka Str., Minsk, 220013; [c]A. D. Sakharov International State Ecological University, 23 Dolgobrodskaya Str., Minsk, 220070; [d]Belarusian State University, 4 Nezavisimost' Ave., Minsk, 220030, Belarus. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 82, No. 4, pp. 810–817, July–August, 2009. Original article submitted April 18, 2008.

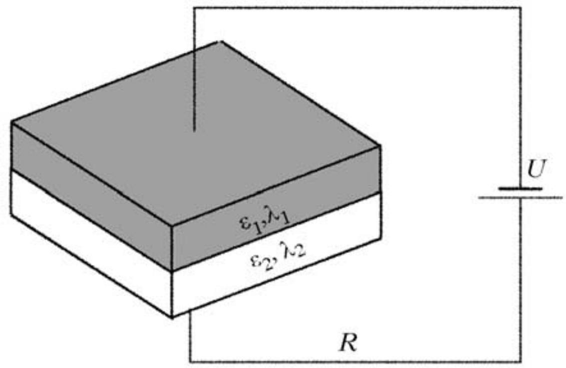


Fig. 1. Dielectric media inside a flat capacitor.

It follows from (3) that the charge σ is determined by the current and the multiplier accounting for the properties of the medium. If

$$(\varepsilon_1/\lambda_1) - (\varepsilon_2/\lambda_2) = 0 \,, \tag{4}$$

a surface charge σ is not formed. What is more, recent trends are toward increased use of micromachines and engines made from plastic materials, where the appearance of a surface charge is undesirable. For oiling of elements of such machines, it is best to use an oil with a permittivity ε_{oil} satisfying the relation

$$\varepsilon_1 < \varepsilon_{\text{oil}} < \varepsilon_2 \,. \tag{5}$$

This oil makes it possible to decrease the electrization of the moving machine parts made from dielectric materials. In addition to the charge σ, a contact potential difference arises always independently of the current.

An electric field interacting with a materials is investigated with the use of the Maxwell equation (1857)

$$\mathbf{j}_{\text{total}} = \nabla \times \mathbf{H} \,, \quad \nabla \cdot \mathbf{D} = \rho \,; \tag{6}$$

$$-\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{E} \,, \quad \nabla \cdot \mathbf{B} = 0 \,, \tag{7}$$

where $\mathbf{j}_{\text{total}} = \lambda \mathbf{E} + \frac{\partial \mathbf{D}}{\partial t}$; $\mathbf{B} = \mu\mu_0 \mathbf{H}$; $\mathbf{D} = \varepsilon\varepsilon_0 \mathbf{E}$. In this case, at the interface S the above system of equations is supplemented with the boundary conditions [1, 2]

$$D_{n_1} - D_{n_2} = \sigma \,, \tag{8}$$

$$E_{\tau_1} - E_{\tau_2} = 0 \,, \tag{9}$$

$$B_{n_1} - B_{n_2} = 0 \,, \tag{10}$$

$$\mathbf{H}_{\tau_1} - \mathbf{H}_{\tau_2} = (\mathbf{i}_\tau \times \mathbf{n}) \,. \tag{11}$$

The indices n and τ denote the normal and tangential components of the vectors to the surface S, and the indices 1 and 2 denote the adjacent media with different electrophysical properties. The index τ denotes any direction tangential to the discontinuity surface. At the same time, a closing relation is absent for the induced surface charge σ, which generates a need for the introduction of an impedance matrix [1–23] that is determined experimentally or, in some cases, theoretically from the quantum representations [5, 10, 21–23].

The induced surface charge σ not only characterizes the properties of a surface, but also represents a function of the process, i.e., $\sigma(\mathbf{E}(\partial\mathbf{E}/\partial t,\ \mathbf{H}(\partial\mathbf{H}/\partial t)))$; therefore, the surface impedances [1–24] are true for the conditions under which they are determined. These impedances cannot be used in experiments conducted under other experimental conditions.

We will show that σ can be calculated using the Maxwell phenomenological macroscopic electromagnetic equations and the electric-charge conservation law accounting for the characteristics of the interface between the adjacent media.

Physical and Mathematical Model. We will formulate a physical and mathematical model of propagation of an electromagnetic field in a layered medium. Let us multiply the left and right sides of the equation for the total current (6) by $\mu_0\mu$ and differentiate it with respect to time. Acting by the operator rot on the left and right sides of the first equation of (7) on condition that μ = const we obtain

$$\frac{\partial \mathbf{j}_{\text{total}}}{\partial t} = \frac{1}{\mu}\nabla^2\mathbf{E} - \frac{1}{\mu}\,\text{grad}\,(\text{div}\,\mathbf{E})\,. \tag{12}$$

In Cartesian coordinates, (12) will take the form

$$\frac{\partial \mathbf{j}_{\text{total}x}}{\partial t} = \frac{1}{\mu}\left(\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2}\right) - \frac{1}{\mu}\frac{\partial}{\partial x}\left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right), \tag{13}$$

$$\frac{\partial \mathbf{j}_{\text{total}y}}{\partial t} = \frac{1}{\mu}\left(\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2}\right) - \frac{1}{\mu}\frac{\partial}{\partial y}\left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right), \tag{14}$$

$$\frac{\partial \mathbf{j}_{\text{total}z}}{\partial t} = \frac{1}{\mu}\left(\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2}\right) - \frac{1}{\mu}\frac{\partial}{\partial z}\left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right). \tag{15}$$

At the interface, the following relation [2] is also true:

$$\text{div}\,\mathbf{i}_\tau + I_{qx_1} - I_{qx_2} = -\frac{\partial\sigma}{\partial t}\,. \tag{16}$$

Let us write conditions (8)–(11) in the Cartesian coordinate system:

$$D_{x_1} - D_{x_2} = \sigma\,, \tag{17}$$

$$E_{y_1} - E_{y_2} = 0\,, \tag{18}$$

$$E_{z_1} - E_{z_2} = 0\,, \tag{19}$$

$$B_{x_1} - B_{x_2} = 0\,, \tag{20}$$

$$H_{y_1} - H_{y_2} = i_z\,, \tag{21}$$

$$H_{z_1} - H_{z_2} = i_y\,, \tag{22}$$

where $\mathbf{i}_\tau = i_y\mathbf{j} + i_z\mathbf{k}$ is the surface-current density, and the coordinate x is directed along the normal to the interface. The densities i_y and i_z of the surface currents represent the electric charge carried in unit time by a segment of unit length positioned on the surface drawing the current perpendicularly to its direction.

The order of the system of differential equations (13)–(15) is equal to 18. Therefore, at the interface S, it is necessary to set, by and large, nine boundary conditions. Moreover, the three additional conditions (17), (21), and (22) containing (prior to the solution) unknown quantities should be fulfilled at this interface. Consequently, the total number of conjugation conditions at the boundary S should be equal to 12 for a correct solution of the problem.

Differentiating expression (17) with respect to time and using relation (16), we obtain the following condition for the normal components of the total current at the medium-medium interface:

$$\operatorname{div}\mathbf{i}_\tau + \mathbf{j}_{\mathrm{total}x_1} = \mathbf{j}_{\mathrm{total}x_2}\,, \tag{23}$$

that allows one to disregard the surface charge σ. Let us introduce the arbitrary function f: $[f]\,|_{x=\xi} = f_1\,|_{x=\xi+0} - f_2\,|_{x=\xi-0}$. In this case, expression (23) will take the form

$$[\operatorname{div}\mathbf{i}_\tau + \mathbf{j}_{\mathrm{total}x}]\,|_{x=\xi} = 0\,. \tag{24}$$

It is assumed that, at the medium-medium interface, E_x is a continuous function of y and z. Then, differentiating (23) with respect to y and z, we obtain

$$\left.\left[\frac{\partial}{\partial y}\mathbf{j}_{\mathrm{total}x}\right]\right|_{x=\xi} = -\frac{\partial\,(\operatorname{div}\mathbf{i}_\tau)}{\partial y}\,, \tag{25}$$

$$\left.\left[\frac{\partial}{\partial z}\mathbf{j}_{\mathrm{total}x}\right]\right|_{x=\xi} = -\frac{\partial\,(\operatorname{div}\mathbf{i}_\tau)}{\partial z}\,. \tag{26}$$

Let us differentiate conditions (20)–(22) for the magnetic induction and the magnetic-field strength with respect to time. On condition that $\mathbf{B} = \mu\mu_0\mathbf{H}$,

$$\left.\left[\frac{\partial B_x}{\partial t}\right]\right|_{x=\xi} = 0\,, \quad \left.\left[\frac{1}{\mu\mu_0}\frac{\partial B_y}{\partial t}\right]\right|_{x=\xi} = \frac{\partial i_z}{\partial t}\,, \quad \left.\left[\frac{1}{\mu\mu_0}\frac{\partial B_z}{\partial t}\right]\right|_{x=\xi} = \frac{\partial i_y}{\partial t}\,. \tag{27}$$

Using Eq. (7) and expressing (27) in terms of projections of the electric-field rotor, we obtain

$$[\operatorname{rot}_x\mathbf{E}]\,|_{x=\xi} = 0 \quad \text{and} \quad \left.\left[\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right]\right|_{x=\xi} = 0\,, \tag{28}$$

$$\left.\left[\frac{1}{\mu\mu_0}\operatorname{rot}_y\mathbf{E}\right]\right|_{x=\xi} = \frac{\partial i_z}{\partial t} \quad \text{or} \quad \left.\left[\frac{1}{\mu\mu_0}\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)\right]\right|_{x=\xi} = \frac{\partial i_z}{\partial t}\,, \tag{29}$$

$$\left.\left[\frac{1}{\mu\mu_0}\operatorname{rot}_z\mathbf{E}\right]\right|_{x=\xi} = \frac{\partial i_z}{\partial t} \quad \text{or} \quad \left.\left[\frac{1}{\mu\mu_0}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\right]\right|_{x=\xi} = \frac{\partial i_z}{\partial t}\,. \tag{30}$$

Here, (28) is the normal projection of the electric-field rotor, (29) is the tangential projection of the rotor on y, and (30) is the rotor projection on z.

Assuming that E_y and E_z are continuous differentiable functions of the coordinates y and z, from conditions (18) and (19) we find

$$\left[\frac{\partial E_y}{\partial y}\right]\Bigg|_{x=\xi}=0\,,\quad \left[\frac{\partial E_y}{\partial z}\right]\Bigg|_{x=\xi}=0\,; \tag{31}$$

$$\left[\frac{\partial E_z}{\partial y}\right]\Bigg|_{x=\xi}=0\,,\quad \left[\frac{\partial E_z}{\partial z}\right]\Bigg|_{x=\xi}=0\,. \tag{32}$$

In accordance with the condition that the tangential projections of the electric field on z and y are equal and in accordance with conditions (18) and (19), the expressions for the densities of the surface currents i_z and i_y take the form

$$i_z=\bar{\lambda}E_z\,|_{x=\xi}\,,\quad i_y=\bar{\lambda}E_y\,|_{x=\xi}\,, \tag{33}$$

where

$$\bar{\lambda}=\frac{1}{2}(\lambda_1+\lambda_2)\,|_{x=\xi} \tag{34}$$

is the average value of the electrical conduction at the interface between the adjacent media in accordance with the Dirichlet theorem for a piecewise-smooth, piecewise-differentiable function.

With allowance for the foregoing we have twelve conditions at the interface between the adjacent media that are necessary for solving the complete system of equations (13)–(15):

a) the functions E_y and E_z are determined from Eqs. (18) and (19);

b) E_x is determined from condition (24);

c) the values of $\partial E_x/\partial y$, $\partial E_x/\partial z$, and $\partial E_x/\partial x$ are determined from relations (25) and (26) with the use of the condition of continuity of the total-current normal component at the interface (24) and the continuity of the derivative of the total current with respect to the coordinate x;

d) the values of $\partial E_y/\partial y$, $\partial E_y/\partial z$, and $\partial E_z/\partial z$ are determined from conditions (31) and (32) in consequence of the continuity of the tangential components of the electric field along y and z;

e) the derivatives $\partial E_y/\partial x$ and $\partial E_z/\partial x$ are determined from conditions (29) and (30) as a consequence of the equality of the tangential components of the electric-field rotor along y and z.

Note that condition (23) was used by us in [25] in the numerical simulation of the pulsed electrochemical processes in the one-dimensional case. Condition (28) for the normal component of the electric-field rotor represents a linear combination of conditions (31) and (32); therefore, $\mathrm{rot}_x E = 0$ and there is no need to use it in the subsequent discussion. The specificity of the expression for the general law of electric-charge conservation at the interface is that the components $\partial E_y/\partial y$ and $\partial E_z/\partial z$ are determined from conditions (31) and (32) that follow from the equality and continuity of the tangential components E_y and E_z at the boundary S.

Thus, at the interface between the adjacent media the following conditions are fulfilled: the equality of the total-current normal components; the equality of the tangential projections of the electric-field rotor; the electric-charge conservation law; the equality of the electric-field tangential components and their derivatives in the tangential direction; the equality of the derivatives of the total-current normal components in the direction tangential to the interface between the adjacent media, determined with account for the surface currents and without explicit introduction of a surface charge. They are true at each cross section of the sample being investigated.

Features of Calculation of the Propagation of Electromagnetic Waves in Layered Media. The electromagnetic effects arising at the interface between different media under the action of plane electromagnetic waves have a profound impact on the equipment because all the real means are fenced in surfaces and are inhomogeneous in the space. At the same time, the study of the propagation of waves in layered conducting media and, according to [26, p. 687–689], in thin films is reduced to the calculation of the reflection and transmission coefficients; the function $E(x)$ is not determined in the thickness of a film, i.e., the geometrical-optics approximation is used.

The physical and mathematical model proposed allows one to investigate the propagation of an electromagnetic wave in a layered medium without recource to the assumptions used in [1–26].

Since conditions (23)–(32) are true at each cross section of a layered medium, we will use schemes of through counting without an explicit definition of the interface between the media. In this case, it is proposed to calculate E_{x_i} at the interface in the following way.

In accordance with (17), $E_{x_1} \neq E_{x_2}$, i.e., $E_x(x)$ experiences a discontinuity of the first kind. Let us determine the strength of the electric field at the discontinuity point $x = \xi$ on condition that $E_x(x)$ is a piecewise-smooth, piecewise-differentiable function having finite one-sided derivatives $E'_{x^+}(x)$ and $E'_{x^-}(x)$. At the discontinuity points x_i,

$$E'^{+}_{x}(x_i) = \lim_{\Delta x_i \to +0} \frac{E(x_i + \Delta x_i) - E(x_i + 0)}{\Delta x_i}, \tag{35}$$

$$E'^{-}_{x}(x_i) = \lim_{\Delta x_i \to -0} \frac{E(x_i + \Delta x_i) - E(x_i - 0)}{\Delta x_i}. \tag{36}$$

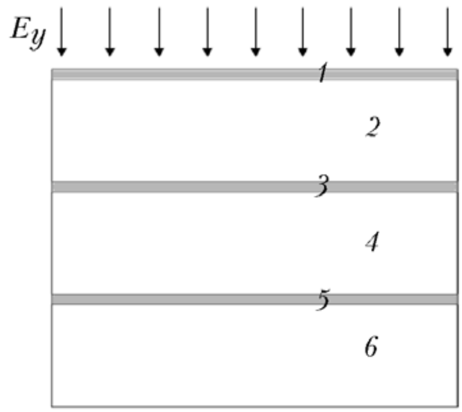
In this case, in accordance with the Dirichlet theorem [27, p. 255–256], the Fourier series of the function $E(x)$ at each point x, including the discontinuity point ξ, converges and its sum is equal to

$$E_{x=\xi} = \frac{1}{2}\left[E(\xi - 0) + E(\xi + 0)\right]. \tag{37}$$

The Dirichlet condition (37) also has a physical meaning. In the case of contact of two solid conductors, e.g., dielectrics or electrolytes in different combinations (metal–electrolyte, dielectric–electrolyte, metal–vacuum, and so on), at the interface between the adjacent media there always arises an electric double layer (EDL) with an unknown (as a rule) structure that, however, substantially influences the electrokinetic effects, the rate of the electrochemical processes, and so on. It is significant that, in reality, the electrophysical characteristics λ, ε, and $E(x)$ change uninterruptedly in the electric double layer; therefore, (37) is true for the case where the thickness of the electric double layer, i.e., the thickness of the interphase boundary, is much smaller than the characteristic size of a homogeneous medium. In a composite, e.g., in a metal with embedments of dielectric balls, where the concentration of both components is fairly large and their characteristic sizes are small, the interphase boundaries can overlap and condition (37) can break down.

If the thickness of the electric double layer is much smaller than the characteristic size L of an object, (37) also follows from the condition that $E(x)$ changes linearly in the EDL region. In reality, the thickness of the electric double layer depends on the kind of contacting materials and can comprise several tens of angströms [28, p. 239]. In accordance with the modern views, the outer coat of the electric double layer consists of two parts, the first of which is formed by the ions immediately attracted to the surface of the metal (a "dense" or a "Helmholtz" layer of thickness h), and the second is formed by the ions separated by distances larger than the ion radius from the surface of the layer, and the number of these ions decreases as the distance between them and the interface (the "diffusion layer") increases. The distribution of the potential in the dense and diffusion parts of the electric double layer is exponential in actual practice [28], i.e., the condition that $E(x)$ changes linearly breaks down; in this case, the sum of the charges of the dense and diffusion parts of the outer coat of the electric double layer is equal to the charge of its inner coat (the metal surface). However, if the thickness of the electric double layer h is much smaller than the characteristic size of an object, the expansion of $E(x)$ into a power series is valid and one can restrict oneself to the consideration of a linear approximation. In accordance with the more general Dirichlet theorem (1829), a knowledge of this function in the EDL region is not necessary to substantiate (37). Nonetheless, the above-indicated physical features of the electric double layer lend support to the validity of condition (37).

The condition at interfaces, analogous to (37), has been obtain earlier [29, p. 353] for the potential field (where rot $\mathbf{E} = 0$) on the basis of introduction of the surface potential, the use of the Green formula, and the consideration of the discontinuity of the potential of the double layer. In [29, p. 356], it is also noted that the consideration



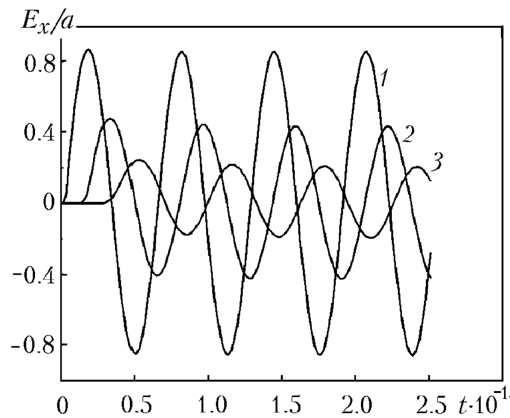


Fig. 2. Scheme of a layered medium: layers 1, 3, and 5 are characterized by the electrophysical parameters ε_1, λ_1, and μ_1, and layers 2, 4, and 6 — by ε_2, λ_2, μ_2.

Fig. 3. Time change in the tangential component of the electric-field strength at a distance of 1 μm (1), 5 μm (2), and 10 μm (3) from the surface of the medium at $\lambda_1 = 100$, $\lambda_2 = 1000$, $\varepsilon_1 = \varepsilon_2 = 1$, $\mu_1 = \mu_2 = 1$, and $\omega = 10^{14}$ Hz. t, sec.

of the thickness of the double layer and the change in its potential at $h/L << 1$ makes no sense in general; therefore, it is advantageous to consider, instead of the volume potential, the surface potential of any density. Condition (37) can be obtained, as was shown in [27], from the more general Dirichlet theorem for a nonpotential vorticity field [29].

Thus, the foregoing and the validity of conditions (17)–(19) and (25)–(32) at each cross section of a layered medium show that, for numerical solution of the problem being considered it is advantageous to use schemes of through counting and make the discretization of the medium in such a way that the boundaries of the layers have common points.

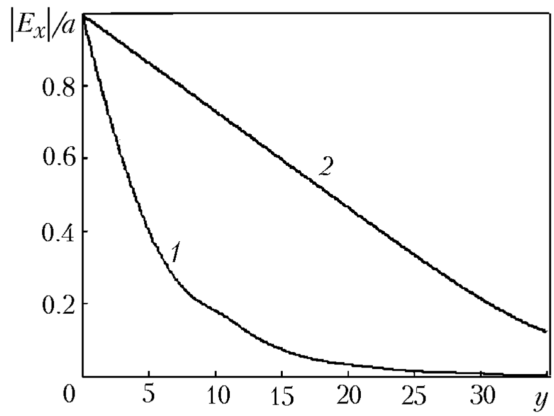
The medium was divided into finite elements so that the nodes of a finite-element grid, lying on the separation surface between the media with different electrophysical properties, were shared by these media at a time. In this case, the total currents or the current flows at the interface should be equal if the Dirichlet condition (37) is fulfilled.

Results of Numerical Simulation of the Propagation of Electromagnetic Waves in Layered Media. Let us analyze the propagation of an electromagnetic wave through a layered medium that consists of several layers with different electrophysical properties in the case where an electromagnetic-radiation source is positioned on the upper plane of the medium. It is assumed that the normal component of the electric-field vector $E_x = 0$ and its tangential component $E_y = a \sin(\omega t)$, where a is the electromagnetic-wave amplitude (Fig. 2).

In this example, for the purpose of correct specification of the conditions at the lower boundary of the medium, an additional layer is introduced downstream of layer 6; this layer has a larger conductivity and, therefore, the electromagnetic wave is damped out rapidly in it. In this case, the condition $E_y = E_z = 0$ can be set at the lower boundary of the medium. The above manipulations were made to limit the size of the medium being considered because, in the general case, the electromagnetic wave is attenuated completely at an infinite distance from the electromagnetic-radiation source.

Numerical calculations of the propagation of an electromagnetic wave in the layered medium with electrophysical parameters $\varepsilon_1 = \varepsilon_2 = 1$, $\lambda_1 = 100$, $\lambda_2 = 1000$, and $\mu_1 = \mu_2 = 1$ were carried out. Two values of the cyclic frequency $\omega = 2\pi/T$ were used: in the first case, the electromagnetic-wave frequency was assumed to be equal to $\omega = 10^{14}$ Hz (infrared radiation), and, in the second case, the cyclic frequency was taken to be $\omega = 10^9$ Hz (radiofrequency radiation).

As a result of the numerical solution of the system of equations (13)–(15) with the use of conditions S (24)–(34) at the interfaces, we obtained the time dependences of the electric-field strength at different distances from the surface of the layered medium (Fig. 3).



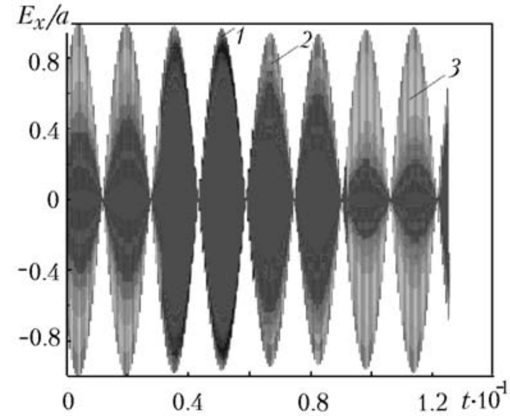


Fig. 4. Distribution of the amplitude of the electric-field-strength at the cross section of the layered medium: $\omega = 10^{14}$ (1) and 10^9 Hz (2). y, μm.

Fig. 5. Time change in the electric-field strength at a distance of 1 (1), 5 (2), and 10 μm (3) from the surface of the medium. t, sec.

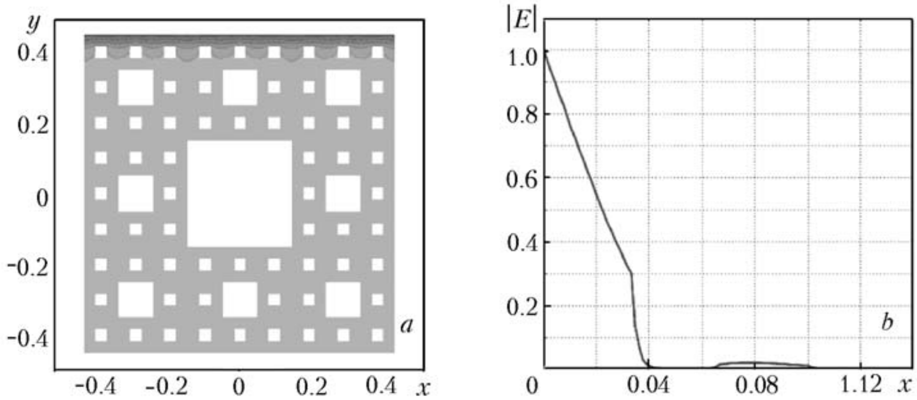


Fig. 6. Distribution of the amplitude of the electric-field strength in the two-dimensional medium (a) and in depth (b) at $\varepsilon_1 = 15$, $\varepsilon_2 = 20$, $\lambda_1 = 10^{-6}$, $\lambda_2 = 10$, $\mu_1 = \mu_2 = 1$, and $\omega = 10^9$ Hz (the dark background denotes medium 1, and the light background — medium 2). x, y, mm; E, V/m.

The results of our simulation (Fig. 4) have shown that a high-frequency electromagnetic wave propagating in a layered medium is damped out rapidly, whereas, a low-frequency electromagnetic wave penetrates into such a medium to a greater depth. The model developed was also used for calculating the propagation of a modulated signal of frequency 20 kHz in a layered medium. As a result of our simulation (Fig. 5), we obtained changes in the electric-field strength at different depths of the layered medium, which points to the fact that the model proposed can be used to advantage for calculating the propagation of polyharmonic waves in layered media; such a calculation cannot be performed on the basis of the Helmholtz equation.

The physical and mathematical model developed can be also used to advantage for simulation of the propagation of electromagnetic waves in media with complex geometric parameters and large discontinuities of the electromagnetic field (Fig. 6).

Figure 6a shows the cross-sectional view of a cellular structure representing a set of parallelepipeds with different cross sections in the form of squares. The parameters of the materials in the large parallelepiped are denoted by index 1, and the parameters of the materials in the small parallelepipeds (the squares in the figure) are denoted by index 2.

An electromagnetic wave propagates in the parallelepipeds (channels) in the transverse direction. It is seen from Fig. 6b that, in the cellular structure there are "silence regions," where the amplitude of the electromagnetic-wave strength is close to zero, as well as inner regions where the signal has a marked value downstream of the "silence" zone formed as a result of the interference.

Conclusions. We were the first to construct a consistent physical and mathematical model of propagation of electromagnetic waves in layered media without recourse to the matrices of the induced-surface-charge impedances. This model is based on the Maxwell equations, the electric-charge conservation law, the total-current continuity, and the Dirichlet theorem. Our numerical investigations have shown that the physical and mathematical model proposed can be used to advantage for simulation of the propagation of a high-frequency electromagnetic wave in a medium consisting of layers having different electrophysical properties.

The authors express their gratitude to Corresponding Member N. V. Pavlyukevich for a useful discussion of the work.

This work was carried out with financial support from the Belarusian Republic Basic Research Foundation (grant T06P-089).

NOTATION

B, magnetic induction, Wb/m^2; **D**, electric displacement, C/m^2; **E**, electric-field strength, V/m; **H**, magnetic-field strength, A/m; h, thickness of the electric double layer, m; I_{qx_1}, I_{qx_2}, normal components of the conduction current in media 1 and 2, C/(m^2·sec); $\mathbf{i}_\tau$, surface current, A/m; **i**, **j**, **k**, unit vectors of the orthonormal basis; $\mathbf{j}_{\text{total}}$, total current, A/m^2; L, size of a sample, m; **n**, unit vector normal to the separation surface; R, resistance, Ω; S, interface between the adjacent media; T, period of oscillations, sec; t, time, sec; U, voltage, V; x, y, z, Cartesian coordinates; ε, relative permittivity; ε_0, electric constant, $8.58 \cdot 10^{-12}$ F/m; λ, conductivity, Ω·m; $\bar{\lambda}$, average conductivity, Ω·m; μ, relative magnetic permeability; μ_0, magnetic constant, $4\pi \cdot 10^{-7}$ H/m; ξ, discontinuity point; ρ, specific electric charge, C/m^2; σ, density of the surface charge, C/m^2; $\nabla \equiv \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$; $[f]|_{x=\xi} \equiv f_1|_{x=\xi+0} - f_2|_{x=\xi-0}$. Subscripts: 1, the first medium; 2, the second medium; n, τ, directions normal and tangential to the separation surface; x, normal component of a vector; y, z, tangential components of a vector at the interface between the adjacent media; oil, oil; total, total; i, number of a grid node.

REFERENCES

1. I. I. Monzon, T. Yonte, and L. L. Sanchez-Soto, Characterizing the reflectance of periodic lasered media, *Opt. Commun*, **218**, 43–47 (2003).
2. Y. Eremin and T. Wriedt, Large dielectric non-spherical particle in an evanescent wave field near a plane surface, *Opt. Commun*, **214**, 34–45 (2002).
3. D. H. Sedrakin, A. H. Gevorgyan, and A. Zh. Khachatrian, Transmission of plane electromagnetic wave obliquely incident on a one-dimensional isotropic dielectric medium with an arbitrary reflective index, *Opt. Commun*, **195**, 35–56 (2001).
4. A. M. Dykhne and I. M. Kasanova, The Leontovich boundary conditions and calculations of effective impedance of homogeneous metal, *Opt. Commun*, **206**, 35–56 (2002).
5. O. Barta, I. Pistora, I. Vesec, et al., Magneto-optics in bi-gyrotropic garnet waveguide, *Opto-Electron. Rev.*, **9**, No. 3, 320–325 (2001).
6. V. P. Red'ko, A. V. Khomchenko, and V. A. Érevich, Automodulation of laser radiation reflected from a two-cavity resonance medium, *Dokl. Akad. Nauk Belarusi*, **47**, No. 1, 57–61 (2003).
7. A. O. Kas'yanov and V. A. Obukhovets, Intellectual radioelectronic coatings, *State-of-the Art and Problems*, Issue 4(5), 4–11 (2001).
8. Wei Hu and Hong Guo, Ultrashort pulsed Bessel beams and spatially induced group-velocity dispersion, *J. Opt. Soc. Am. B*, **19**, No. 1, 49–52 (2002).

9. Delbeke Danae, P. Bienstman, R. Bockstaele, and R. Baets, Rigorous electromagnetic analysis of dipole emission in periodically corrugated layers: the grating-assisted resonant-cavity light-emitting diode, *J. Opt. Soc. Am. B,* **19**, No. 5, 871–881 (2002).
10. I. Broe and O. Keller, Quantum-well enhancement of the Goos–Hanchen shift for p-polarized beams in a two-prism configuration, *J. Opt. Soc. Am. B,* **19**, No. 6, 1212–1221 (2002).
11. J. I. Larruquert, Reflectance enhancement with sub-quarterwave multilayers of highly absorbing materials, *J. Opt. Soc. Am. B*, **18**, No. 6, 1406–1415 (2001).
12. I. Simonsen, D. Vanderbrouoq, and S. Roux, Electromagnetic wave scattering from conducting self-affine surfaces: an analytic and numerical study, *J. Opt. Soc. Am. B*, **18**, No. 5, 1101–1111 (2001).
13. J. M. Bendikson., E. N. Glytsis, T. K. Gaslord, and A. F. Peterson, Modeling considerations for rigorous boundary element method analysis of diffractive optical elements, *J. Opt. Soc. Am. B*, **18**, No. 7, 1495–1506 (2001).
14. K. A. O'Donnell, High-order perturbation theory for light scattering from a rough metal surface, *J. Opt. Soc. Am. B*, **18**, No. 7, 1507–1516 (2001).
15. B. M. Koludzija, Electromagnetic modeling of composite metallic and dielectric structures, *IEEE Trans. Microwave Theory Tech.*, **47**, No. 7, 1021–1029 (1999).
16. R. A. Ehlers and A. C. Metaxas, 3-DFE Discontinuous sheet for microwave heating, *IEEE Trans. Microwave Theory Tech.*, **51**, No. 3, 718–726 (2003).
17. T. Tischler and W. Heinrich, The perfectly matched layer as a lateral boundary in finite-difference transmission-line analysis, *J. Opt. Soc. Am. A*, **48**, No. 12, 2249–2257 (2000).
18. M. K. Karkkainen and S. A. Tretyakov, A class of analytical absorbing boundary conditions originating from the exact surface impedance boundary conditions, *J. Opt. Soc. Am. A*, **51**, No. 2, 561–577 (2003).
19. F. L. Teixeira, C. D. Moss, W. C. Chewand, and Jim A. Kong, Split-field and anisotropic-medium PML–FDID implementations for inhomogeneous media, *J. Opt. Soc. Am. A*, **50**, No. 1, 31–38 (2002).
20. Le-Weili, Xiao-Kang Hang, Pans-Shyan Kooi, and Tat-Soonyeo, Electromagnetic dyadic Green's functions for multilayered spheroidal structures-1: Formulation, *J. Opt. Soc. Am. A*, **49**, N. 3, 532–541 (2001).
21. O. Keller, Optical response of a quantum-well sheet: internal electrodynamics, *J. Opt. Soc. Am. B*, **12**, No. 6, 997–1005 (1995).
22. O. Keller, Sheet-model description of the linear optical response of quantum wells, *J. Opt. Soc. Am. B*, **12**, No. 6, 987–997 (1995).
23. O. Keller, Local fields in linear and nonlinear optics of mesoscopic system, *Prog. Opt.*, **37**, 257–343 (1997).
24. G. D. Gol'dshtein and N. V. Zernov, *Electromagnetic Fields and Waves* [in Russian], Nauka, Moscow (1971).
25. N. N. Grinchik and A. P. Dostanko, *Influence of Thermal and Diffusional Processes on the Propagation of Electromagnetic Waves in Layered Materials* [in Russian], ITMO im. A. V. Luikova, NAN Belarusi, Minsk (2005).
26. M. Born and E. Wolf, *Principles of Optics* [Russian translation], Mir, Moscow (1970).
27. L. D. Kudryavtsev, *Mathematical Analysis* [in Russian], Vol. 2, Mir, Moscow (1970).
28. A. N. Frumkin, *Electrode Processes* [in Russian], Nauka, Moscow (1987).
29. A. N. Tikhonov and A. A. Samarskii, *Equations of Mathematical Physics* [in Russian], Nauka, Moscow (1977).